

A Survey on Convergence of Various Adaptive Algorithms

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ABSTRACT: In many digital signal processing applications extremely used filter is adaptive filter due to self adjusting transfer function. Channel characteristics are often unknown or time variant in environment. In this case, Adaptive filter exhibits accurate signal by adjusting filter coefficient according to adaptive algorithm. This paper describes different types of adaptive algorithms and conditions for convergence of the algorithms. In addition to, Error measurement techniques are also analysed.

KEYWORDS: Adaptive filter, Adaptive algorithms, Error measurement techniques

I. INTRODUCTION

In the last decades, Adaptive signal processing has developed extremely due to the increasingly availability of technology for the implementation of the emerging algorithms. These algorithms has been applied to many problems including noise and echo cancelling [11], channel equalization, signal prediction, adaptive arrays as well as many others[1]. The traditional filters require a priori knowledge about the statistics of the data which is used in process. In practical, Filter is operating in an environment, in which environment is non stationary then filter faces more problems. These problems are not faced in adaptive filter.

The main difference between adaptive filter and traditional filter is traditional filter coefficients are constant irrespective of time but adaptive filter coefficients are changing with respect to the time. The filter coefficients are change in accordance with an algorithm. The algorithm helps to reduce cost function by changing parameters of filter coefficient. The cost function of error signal has been measured by using error measurement techniques. Mostly used techniques are least mean square error, Instantaneous Square Error, Weighted Least Square and correntropy [7]. Depending on applications select the suitable error measurement technique for adaptive algorithm.

A group of algorithms explained in this paper. Each algorithm presents particular characteristics of computational complexity and speed of convergence. This leads to determine the best possible solution to an application. Speed of convergence mainly depends on free parameters (step size, forgetting factor) except in fixed point algorithm (FP) and dynamic harmonic balance algorithm (DHB).

Paper is organized as follows. Section II describes adaptive filter. Different types of adaptive algorithm explained in Section III. Section IV presents different types of error measurements and corresponding functions. Finally, Section V presents conclusion.

II. ADAPTIVE FILTER

Adaptive filter called as self modifying digital filter because filter adjusting its filter coefficients in order to minimize error function. These filter primarily chosen when channel characteristics unknown and channel response is time

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variant. Adaptive filter is a computational device that attempts to models the relationship between the input and output signals in real time in an iterative manner.

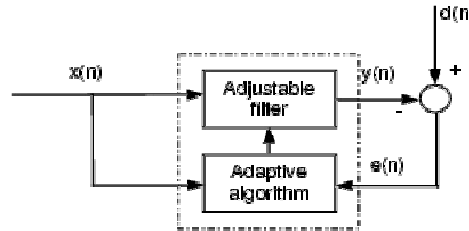


Fig.1 Adaptive Filter

In Fig.1, Input signal $X(n)$ is pass through the adaptive filter, it computes corresponding output signal $Y(n)$. The output signal $Y(n)$ compares with another input signal that is desired signal $d(n)$ by subtracting two signal samples at a time n . The result signal is called error function $e(n)$. The error signal fed into the adaptive algorithm which adapts filter coefficients from time index n to the next time index $(n+1)$. There are many iterative algorithms derived for minimizing error function. Some of the adaptive algorithms are

1. Steepest Descent Algorithm(SD)
2. Least Mean Square (LMS) and Normalised Least Mean Square(NLMS) Algorithm
3. Affine Projection Algorithm (AP)
4. Recursive Least Square Algorithm (RLS)
5. Fixed Point Algorithm(FP)
6. Dynamic Harmonic Balance Algorithm(DHB)

Convergence speed is one of the main characteristics in Adaptive algorithm. The convergence speed is number of iterations to be taken for the algorithm to reach desired state.

III. ADAPTIVE ALGORITHMS

1. Steepest Descent Algorithm (SD)

Steepest descent algorithm is old and deterministic method. The optimized procedure of this algorithm is minimizing the value of error function $J(n)$ with respect to a set of adjustable filter coefficients $W(n)$. The procedure to adjust each filter coefficient in according to

$$W(n+1) = W(n) - \mu(n) \frac{\partial J(n)}{\partial W(n)} \quad (1)$$

Here $\mu(n)$ is Step size and error function is calculated by using Mean square error i.e.

$$J_{MSE}(n) = E[e^2(n)] \quad (2)$$

From the Fig.1 $e(n) = d(n) - Y(n)$ and $Y(n) = W^T X(n)$ Substituting $\frac{\partial J(n)}{\partial W(n)}$ value from the equation (2)

and update the equation (1) as

$$W(n+1) = W(n) + \mu(P_{dx}(n) - R_{xx}(n)W(n)) \quad (3)$$

However SD algorithm depends upon auto correlation $R_{xx} = E[XX^T]$ and cross correlation $P_{dx} = E[dX]$ functions. The algorithm requires priori knowledge about R and P .

Step size always $\mu > 0$. If algorithm take a step to negative direction. The choice of step size is critical. The algorithm may diverge when the steps are too large. The convergence may take more time when the steps are too small. So, step size is taken very carefully that algorithm neither diverges nor more time to converge. i.e.

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$$\mu < \frac{XX^T}{\lambda_{\max}} \tag{4}$$

Where λ_{\max} is maximum Eigen value of auto correlation (R).

2. Least Mean Square(LMS) and Normalized Least Mean Square(NLMS) Algorithm

The LMS algorithm [10] is popular and widely used algorithm. It is one of the stochastic methods because statistics (that is cross correlation (P) and auto correlation(R)) are estimated instantaneously. LMS algorithm just like SD algorithm but based on unknown statistics [9]. Here estimate cross correlation \hat{P} and auto correlation \hat{R} to calculate

$\frac{\partial J(n)}{\partial W(n)}$ and substitute in equation (1). The updated equation is

$$W(n+1) = W(n) + \mu E[X(n)e(n)] \tag{5}$$

The algorithm converges when it satisfies two conditions. The conditions are

1. The auto correlation (R) must be positive definite
2. $0 < \mu < \frac{1}{\lambda_{\max}}$

Convergence speed dependent on condition number of auto correlation (k) is defined as ratio between minimum Eigen values to maximum Eigen value of auto correlation. i.e.

$$k = \frac{\lambda_{\min}}{\lambda_{\max}} \tag{6}$$

NLMS algorithm increase convergence speed than the LMS algorithm. The step size modified as

$$\mu = \frac{\bar{\mu}}{XX^T + \epsilon} \tag{7}$$

Where $\bar{\mu}$ is used to control misadjustment and control speed within range $0 < \bar{\mu} \leq 1$. When $\bar{\mu} = 1$, the fastest convergence attained at maximum misadjustment. ϵ is very small positive integer. It avoids division by zero. In absence of the disturbance step size is in the range of $0 < \mu < 2$ [8].

3. Affine Projection Algorithm (AP)

Affine Projection Algorithm is generalization of NLMS algorithm. The main difference between the algorithms is filter coefficients are updated and viewed as one dimensional in NLMS algorithm. In AP algorithm, the projections are made in multi directions. AP algorithm update filter coefficients based on k (finite number) input vectors. The algorithm adjust filter coefficient as

$$W_{n+1} = W_n + \mu X_n^T (X_n X_n^T)^{-1} e_n \tag{8}$$

Where $X_n = \begin{bmatrix} X_n \\ X_{n-1} \\ \vdots \\ X_{n-k+1} \end{bmatrix}$ and $d_n = \begin{bmatrix} d_n \\ d_{n-1} \\ \vdots \\ d_{n-k+1} \end{bmatrix}$

Algorithm is computationally complex. The performance of convergence also good in colored input data. This algorithm converges when it satisfies two conditions. There are

1. The number of iterations approach to infinite than expectation of filter coefficient approaches to zero
2. The steady state value of covariance $COV(W_k)$ (in detail explanation in [3]) must be finite.

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4. Recursive Least Square Algorithm(RLS)

RLS algorithm belongs to kalman filter family RLS algorithm gives excellent perform when working in time varying environment. This algorithm excellent in Convergence speed. It is independent of the Eigen value of auto correlation function. The main drawback of this algorithm is high computational complexity. RLS algorithm mainly useful in slowly varying environment. The auto correlation and cross correlation function defined as

$$\mathbf{R}(n) = \mathbf{X}(n)\mathbf{X}^T(n) + \lambda\mathbf{R}(n-1) \quad (9)$$

$$\mathbf{P}(n) = \mathbf{X}(n)\mathbf{d}(n) + \lambda\mathbf{P}(n-1) \quad (10)$$

Here λ is forgetting factor [2]. $0 \ll \lambda < 1$

Error function calculated by using weighted least square function. The updated filter coefficients are

$$\mathbf{W}(n) = \mathbf{W}(n-1) + \mathbf{e}(n)\mathbf{R}^{-1}(n)\mathbf{X}(n) \quad (11)$$

Where $\mathbf{R}^{-1}(n) = \frac{1}{\lambda}[\mathbf{R}^{-1}(n-1) - \mathbf{k}'(n)\mathbf{k}^T(n)] \quad (12)$

$$\mathbf{k}'(n) = \mathbf{R}^{-1}(n)\mathbf{X}(n) \quad (13)$$

$$\mathbf{k}(n) = \mathbf{R}^{-1}(n-1)\mathbf{X}(n) \quad (14)$$

From (13) and (14) we reproduced as

$$\mathbf{k}'(n) = \frac{\mathbf{k}(n)}{\lambda + \mathbf{X}^T\mathbf{k}(n)} \quad (15)$$

RLS algorithm convergence rate, misadjustment, tracking and stability are depending on forgetting factor. If forgetting factor equal to 1, It will leads to misadjustment and high stability.

5. Fixed Point Algorithm(FP)

The convergence of fixed point algorithm[5] is extremely fast. FP does not use free parameters like step size, forgetting factor and more. The updated filter coefficients are

$$\mathbf{W}_{k+1} = \mathbf{f}(\mathbf{W}_k) \quad (16)$$

Where

$$\mathbf{f}(\mathbf{W}) = [\mathbf{R}_{XX}^G]^{-1} \mathbf{P}_{dX}^G$$

$$\mathbf{R}_{XX}^G = \frac{1}{N} \sum_{i=1}^N G_{\sigma}(e(i))\mathbf{X}(i)\mathbf{X}(i)^T$$

$$\mathbf{P}_{dX}^G = \frac{1}{N} \sum_{i=0}^N G_{\sigma}(e(i))\mathbf{d}(i)\mathbf{X}(i)$$

Kernel function $k(\mathbf{X}, \mathbf{Y}) = G_{\sigma}(e) = \exp\left(-\frac{e^2}{2\sigma^2}\right)$

Error function is calculated by using correntropy method. Correntropy depends on the value bandwidth σ . FP algorithm converges when it satisfy two conditions. There are

$$1. \|\hat{\mathbf{f}}(\mathbf{W})\|_p \leq \beta \quad (\beta > 0)$$

$$2. \left\| \nabla_w \hat{\mathbf{f}}(\mathbf{w}) \right\|_p = \left\| \frac{\partial \hat{\mathbf{f}}(\mathbf{w})}{\partial \mathbf{w}} \right\|_p \leq \alpha \quad (0 < \alpha < 1)$$

Here β and α are positive and finite value (in detail [5]).

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6. Dynamic Harmonic Balance Algorithm(DHB)

DHB algorithm[6] procedure similar to FP algorithm. The convergence speed of DHB algorithm fast than FP algorithm. The condition for convergence of DHB algorithm is

$$N(a, \sigma)W_\ell(\sigma + j\Omega) = -1 \quad (17)$$

Here $N(a)$ is nonlinear function which depends on amplitude of function a and σ is decay.

IV. ERROR MEASUREMENT

Adaptive filter coefficients updated every time depending on error function. Error function calculated in 4 ways there are mean square error, instantaneous square error, weighted least square function and correntropy.

A. Mean Square error(MSE)

MSE defined as

$$J(n) = E[e^2(n)] = E\left[|d(n) - W^T X(n)|^2\right] \quad (18)$$

Simplifies (18) and rewritten as

$$J(n) = E[d^2(n)] - 2W^T P + W^T R W \quad (19)$$

The derivative of MSE function with respect to the filter coefficient is given by

$$\frac{\partial J}{\partial W} = -2P + 2RW \quad (20)$$

B. The Instantaneous Square Error(ISE)

ISE defined as

$$J(n) = e^2(n) \quad (21)$$

The derivative of ISE with respect to the filter coefficient is given by

$$\frac{\partial J}{\partial W} = -2e(n)X(n) \quad (22)$$

C. The Weighted Least Square(WLS)

WLS defined as

$$J(n) = \sum_{i=0}^n \lambda^{n-i} [d(i) - W^T X(i)]^2 \quad (23)$$

$$\text{Here } X_n = \begin{bmatrix} X_n^T \\ \lambda^{1/2} X_{n-1}^T \\ \vdots \\ \lambda^{n/2} X_{n-k+1}^T \end{bmatrix} \text{ and } d_n = \begin{bmatrix} d_n \\ \lambda^{1/2} d_{n-1} \\ \vdots \\ \lambda^{k/2} d_{n-k+1} \end{bmatrix}$$

From (23) the function depends on several error functions so, we get more time to calculate the function than remaining two functions (MSE and ISE) but noise is reduced. The derivative of ISE with respect to the filter coefficient is given by

$$\frac{\partial J}{\partial W} = -2P + 2RW \quad (24)$$

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D. Correntropy

Correntropy[7] is a nonlinear similarity measured between two random variables. Correntropy is maximum when both random variables X and Y are equal.

Correntropy defined as

$$V(X, Y) = E(k(X, Y)) = \int k(X, Y) dF_{XY}(X, Y) \quad (25)$$

Here $k(X, Y)$ =kernel function

$F_{XY}(X, Y)$ = joint distribution function between X and Y

The derivative of ISE with respect to the filter coefficient is given by

$$\frac{\partial J}{\partial W} = -P + RW \quad (26)$$

Where $P = E[G_{\sigma}(e)dX]$ and $R = E[G_{\sigma}(e)XX^T]$

V. CONCLUSION

In this paper, we have surveyed different types of adaptive algorithms. We analysed the convergence behaviour and corresponding conditions of the algorithms. SD, LMS, NLMS, AP and RLS algorithms convergence speed mainly depend on parameter like step size, forgetting factor etc., It is difficult to chosen optimum value of parameter for convergence the algorithm. This problem overcomes in FP and DHB algorithm. Here also explained different types of techniques for measuring error function.

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